EXTENT OF TRAFFIC KINEMATIC WAVES AND QUEUING CAUSED BY MIDBLOCK U-TURN FACILITIES

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Abstract

The paper is aimed at determining the extent of highway traffic kinematic waves and delays induced by midblock U-turn facilities provided along many dual carriageways in Malaysia. U-turn facilities at signalised and priority intersections are very commonly found along federal highways in Malaysia. However, the introduction of U-turn facilities along highway segments has provoked fierce national debates about their benefits and risks. Often U-turn traffic movements at highway segments are channelized and aided with splitting islands so that drivers can be on their desired trajectories. Drivers will have to keep to the right lane; decelerate when diverging, accelerate when converging. These dangerous manoeuvres beg the question; 'what are the induced traffic flow consequences when the lead vehicle decelerates or accelerates abruptly? Based on the hypothesis that U-turn facilities at highway segments have impact on traffic kinematic wave propagations; impact studies were carried out in Skudai, Malaysia during daylight and dry weather conditions. Traffic volume, speed, headway and vehicle types' were collected continuously for six weeks. The survey data were supplemented with highway design information culled from the Malaysian Public Works Departments manual. Traffic shockwave velocity propagations were estimated for highway sections 'with and without' U-turn facilities and then compared. Results show that significant traffic kinematic shockwave of about 21km/h occurred only when converging. Further, average delay of about 8.5s/veh occurred at the exit lane. The paper concluded that U-turn facilities will induce traffic kinematic waves that may cause road accidents as well as cause delay.

Keywords: U-Turn, midblock, highway, traffic shockwave, road accident

1. INTRODUCTION

In Malaysia, U-turn facilities at signalised and priority intersections are very commonly found along federal highways. Midblock U-turn facilities are often constructed as a cost effective way of alleviating congestion and road safety problems. As contained many literatures, midblock U-turn facilities are effective conflict-points reduction mechanism at intersections. An intersection without treatment has 32 conflict points (16 crossing, 8 diverge, 8 merge), however, at treated intersection conflict points are reduced to 8 (1 crossing, 3 diverge, 4 merge) [9]. The more common right turn treatments used on urban and suburban arterials are: flash median with one way right turn lane, raised curb median with alternating right turn bays, flush median with alternating right turn and undivided cross section as contained in NCHRP Report 395 [3]. One potential treatment to combat congestion and safety problems at intersections is the installation of non-traversable medians and directional median opening has produced an increased number of U-turns on multilane divided roadways. Congestion at the intersections throughout urban and suburban areas continues to worsen; crashes at intersections have continued to increase according to reports in the newspaper. In any case arguments have been advanced by some opponents of median modification projects that the increased numbers of U-turns may result in safety and operational problems on multilane divided roadway. Often U-turn traffic movements at highway segments are channelized and aided with splitting islands so that drivers can be on their desired trajectories. Drivers will have to keep to the right lane; decelerate when diverging, accelerate when converging. These dangerous manoeuvres beg the question; 'what are the induced traffic flow and safety consequences when the lead vehicle decelerates or accelerates abruptly? The aim of the study is to determine the extent of traffic kinematic waves caused by midblock U-turn facilities. The primary objective of this study is to use flow/density function to determine flowrate contractions for carriageway lane 'with and without' U-turn facility under dry weather and daylight conditions.

Malaysia consists of thirteen states and three federal territories and has a total landmass of 329,847m^2 separated by the South China Sea into two similarly sized regions, Peninsular Malaysia and Malaysian Borneo. The capital city is Kuala Lumpur. In 2010 the population exceeded 27.5 million, with over 20 million living on the Peninsula. Malaysian highways are classified by the ministry of works as expressway, federal, state, municipal highways and others. The expressways are defined as high-speed routes with at least four lanes (2 lanes per carriageway) with full access control, grade-separated interchanges and high design speed limit of 120 km/h, allowing the maximum speed limit of 110 km/h. It has a total length of about 1,850km [2]. Federal Highways connect all state capitals and city of Kuala Lumpur. It is the busiest highway. They are often built with 2 carriageway lanes in each direction and an operating speed limit of 90km/h. Motorists are expected to travel faster when overtaking on right lane. State highways connect district headquarters and are normally single carriageway road. Municipal highways connect residential, commercial and other roads within their district of influence. Midblock U-turn facilities are commonly found along federal roadways in Malaysia. There are mainly two types: dedicated right U-turn and open right U-turn facilities. Dedicated right U-turn facility has grade separated lane whereas open right U-turn facility allows motorists to diverge or merge with through traffics. The introduction of U-turn facilities along roadway segments has provoked fierce national debates about their benefits and risks.

2. KINEMATIC WAVE THEORY OF TRAFFIC FLOW IN CONTEXT

The fundamental traffic theory states that: i), at any point of the road the flow is a function of the concentration; ii), it is compatible with or dependent on fundamental diagram of traffic; iii) It has 4 basic boundary conditions: iv), flow = zero when density is zero; iv), flow = zero when density is at jam, v), speed = zero at jam density, vi), speed ≡ free flow when density is zero. Flow-density curve are often described in many literatures as parabolic or asymmetric shaped. If the interest is empirical capacity estimation, the parabolic curve will suffice since the main interest in the ensuing curve is the vertex point (Q) as shown below in figure 1. However if the interest is a curve that fulfills the basic theory of traffic then asymmetric curve will describe more accurately the real traffic stream scenario because it consists of two vectors (free-flow and congested). Speed oscillates within the free-flow sector; whereas flowrate contracts in the congested sector. Further, this sector has a negative slope, which implies that the higher the density, the lower the flow; in cases where there are more cars on the roadway, the number of cars passing a single point is less than if there were fewer cars on the road.

As shown in figure 1, speed (ϴ) will oscillate from point v to Q; beyond Q oscillation will cease because traffic flow is now constrained by roadway capacity (Q). Should the lead driver brake abruptly due to changes in traffic, roadway, weather or ambient conditions the resultant shockwave will be along lines Q_1 Q_2 and Q_3. In essence a driver experiences kinematic wave whenever he/she adjusts his/her speeds in accordance with the behaviour of the car or cars in front, on observing a brake light, or an opportunity to overtake. Kinematic waves can run together to form 'kinematic shock waves', at which fairly large reductions in velocity occur very quickly. These are far too common on highways with midblock U-turn movements. Shockwaves are by-products of traffic congestion [4]. They are transition zones between two contrasting traffic states (unconstrained and constrained traffic flows.). On the urban freeway, most drivers can identify them as a transition from a flowing, speedy state to a congested, standstill state. However, shockwaves are also present in the opposite case, where drivers who are idle in traffic suddenly are able to accelerate. Shockwaves are one of the major safety concerns because the sudden change of conditions drivers experience as they pass through a shockwave often can cause accidents. Note that critical density shifts to the right thus indicating that some vehicles are entering the traffic stream. So, if it is assumed that the coordinates on the curve beyond capacity (Q) depict congestion, it can be postulated correctly that traffic shockwave is function of traffic flowrate contraction.

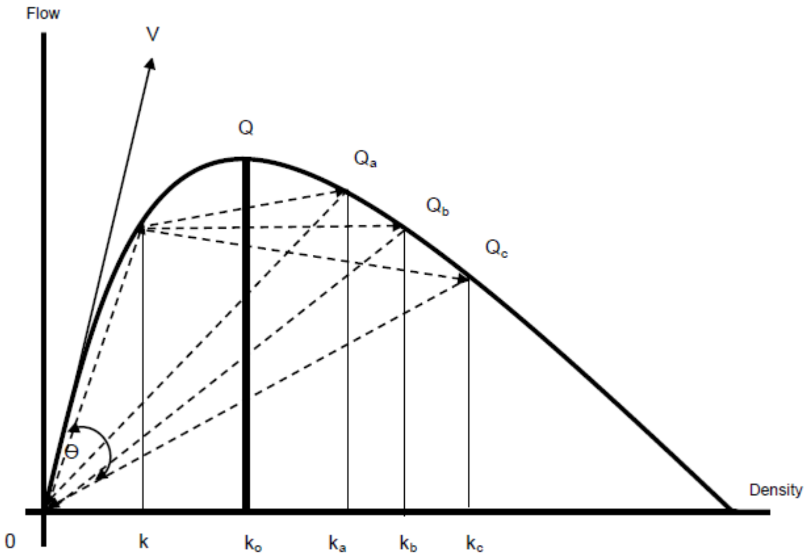

Figure 1 Hypothetical Traffic shockwave and flowrate contraction

Because highway traffic involves flow, density and speed, there is always a tendency to describe traffic in terms of fluid behaviour; unlike fluid is the movement of discrete object in 2D not 1D. Lighthill and Whitham [9] postulated that there exists some functional relationship between flow and density that may vary with location but not with time. The relationship depicted as equation 1, forms the basis for quantitative assessment of traffic flow. Where location is *x* and time is *t*; then

$$k\,(x,t) = k\,(q(x,t),x) \tag{1}$$

If it assumed that there is no vehicle entering or exiting the traffic stream, then the equation of continuity can be applied to equation 1 to give a partial differential equation 2 for q (x, t).

$$\frac{\partial k\,(x,t)}{\partial t} + \frac{\partial k\,(x,t)}{\partial t} = 0 \tag{2}$$

This is an exaggerated assumption nonetheless;

$$S_{w\,(q(x,t),x)}\frac{\partial k\,(x,t)}{\partial t} + \frac{\partial k\,(x,t)}{\partial t} = 0 \tag{3}$$

With

$$S_w(q,x) = \frac{\partial k(q,x)}{\partial q} \tag{4}$$

For $1/s_w$ is traffic shockwave velocity propagation.
Three primary measures namely: flow, speed and density characterise the operational state of any given traffic stream. In theory, where the flow / density relationship has been used to compute roadway capacity according to Ben-Edigbe [1], the critical density is reached at the apex point of flow/density curve and the function can be written as;

$$Q = -c + (v_f)\frac{v_f}{2\left(\frac{v_f}{k_j}\right)} - \frac{v_f}{k_j}\left(\frac{v_f}{2\left(\frac{v_f}{k_j}\right)}\right)^2 \tag{5}$$

Where Q is capacity, c is a constant, v_f is free-flow speed and k_j is jam density

According to Ben-Edigbe [1] where the flow / density relationship has been used to compute roadway capacity where critical density is reached at the apex point. Up till that point, traffic stream is operating under unconstrained conditions not free flow as often wrongly mentioned in many literatures. Beyond the apex point, traffic flowrate is operating under constrained condition. Since our interest is in estimating the traffic kinematic changes due to midblock right u-turning movement, the choice of precise value of critical density need not be very critical to the outcome of this study. Consider Equation 5 again, for maximum flow;

$$\frac{\partial_q}{\partial_k} = u_f - 2\left(\frac{u_f}{k_j}\right)k = 0 \tag{6}$$

Then, critical density

$$k_c = \frac{u_f}{2\left(\frac{u_f}{k_j}\right)} \tag{7}$$

Where k_c is the critical density; as contained in previous studies and many literatures, shockwave equation is;

$$v_w = \frac{q_2 - q_1}{k_2 - k_1} \tag{8}$$

If equation 5 is plugged into equation 8 then, traffic shockwave velocity can be re-written as:

$$v_w = \frac{q_2 - \left\{(u_f)\frac{u_f}{2\left(\frac{u_f}{k_j}\right)} - \frac{u_f}{k_j}\left(\frac{u_f}{2\left(\frac{u_f}{k_j}\right)}\right)^2\right\}}{k_2 - \left\{\frac{u_f}{2\left(\frac{u_f}{k_j}\right)}\right\}} \tag{9}$$

If the assertion that, 'traffic shockwaves are by-products of traffic congestion' is to hold, then q_1 and k_1 must be congested flowrate and density respectively. Therefore, a threshold capacity must be estimated in other to ascertain whether the threshold line has indeed been crossed as shown earlier in figure 1. Where the threshold capacity has been crossed the passenger car equivalent values being an instrument of capacity computation must also be modified. Ignoring PCE modifications could lead to grossly inaccurate road capacity estimates with consequences for road transportation modeling.

Passenger car equivalent (PCE) values are usually relied on to convert vehicle volume into traffic flow be it at junction, roundabout, traffic signals, rural , urban or just a road link section. Since PCE measures the impact that a mode of transport has on traffic variables compared to a passenger car under prevailing conditions, it follows that changes in prevailing conditions will have relative effect on *pce* values. In essence *pce* values are dynamic.

Therefore equation 8 must be used in conjunction with dynamic *pce* values. The term 'passenger car equivalent' was defined in Highway Capacity Manual (HCM) as 'the number of passenger cars displaced in the traffic flow by truck or a bus under the prevailing roadway and traffic conditions'. This definition still holds today and the use of such equivalents is central to road capacity analysis where mixed traffic stream are present.

The headway evaluation criteria could be applied to many traffic situations such as at intersection and basic highway segments or mid-block sections. Whereas headway data can be obtained in the field with relative ease, other evaluation criteria such as delay, density and speed are expensive as such methods based on these adopt the simulation approach. The passenger car equivalency method used in this study is the headway method. The method was first proposed by and involves the following equation;

$$PCE_i = \frac{H_i}{H_c} \tag{10}$$

Where PCE_i is the passenger car unit of vehicle class *i*. H_i is the average headway of vehicle class *i* and H_c is the average headway of passenger car.

2.1 Weaving, Merging, Diverging and Delays

A U-turn refers to vehicles performing a 180 degree rotation to reverse the direction of travel [4]. Midblock right U-turn facilities are built to assist with these deft manoeuvres. While some are built as complimentary facilities to existing road geometric design, others are built as a complete replacement to existing facilities. Interaction between different modes of traffic on the highway is an important issues because drivers alone must decide when it is safe to merge, diverge and accept emerging gaps mindful of intersection priority rules. Weaving, merging and diverging occur at and between midblock U-turn facilities. Often U-turn traffic movements at roadway segments are channelized and aided with splitting islands so that drivers can be on their desired trajectories. Right U-turn movements are channelized and aided with splitting islands so that drivers can be on their desired trajectories.

It can be postulated that right U-turn movements will inherently interfere with through traffic by encroaching on part or all of the through traffic lanes. Drivers will have to keep to the right lane; decelerate when diverging, accelerate when converging. These dangerous manoeuvres beg the question; 'what are the induced traffic flow consequences when the lead vehicle decelerates or accelerates abruptly? On approach to Midblock U-turn facilities, weaving is often inherent because of drivers desire to position their vehicle along the appropriate carriageway lane. When exiting the facility, driver may reject gap on the major road and wait for a subsequent gap. Poor gap acceptance decisions have severe consequences. They may cause shockwave and lead to accidents. The existence of traffic shockwave velocity propagations at the weaving area of the midblock right U-turn facilities is a clear indicator of road accident risk associated with midblock right U-turn facilities.

Although it can be argued that the presence of traffic shockwave cannot be associated solely with midblock U-turn given that traffic shockwave can occur without the presence of midblock right U-turn facilities. That may be true, but it has yet to be substantiated with research evidence. In any case, it important to determine the presence of kinematic waves as well as queue length and delays at midblock U-turn carriageway lane because of safety due to traffic overflow. Many studies have been carried out with respect to capacities and delays at priority intersections, so there is no need to build a new model. The maximum traffic flow from the midblock U-turn carriageway according to Tanner [5] can be estimated as;

$$q_{max} = \frac{q_1(1-\beta_1 q_1)}{\exp[q_1(\alpha-\beta_1][1-\exp(-\beta_2 q_1)]} \tag{11}$$

In addition Tanner [5] derived an expression for average delay to minor road (midblock U-turn lane) shown below:

$$\overline{w_2} = \frac{\frac{1}{2}E(y^2)/Y\ exp\,(-\beta_2 q_1)[\exp(\beta_2 q_1) - \beta_2 q_1 - 1]/q_1}{1 - q_2 Y[1 - \exp(-\beta_2 q_1)]} \tag{12}$$

For;

$$E(y) = \frac{\exp[q_1(\alpha - \beta_1)]}{q_1(1 - \beta_1 q_1)} - \frac{1}{q_1}$$

$$E(y^2) = \frac{2\exp[q_1(\alpha - \beta_1)]}{q_1{}^2(1 - \beta_1 q_1)^2}\{\exp[q_1(\alpha - \beta_1)] - \alpha q_1(1 - \beta_1 q_1) - 1 + \beta_1 q_1 - \beta_1{}^2 q_1{}^2 + \frac{1}{2}\beta_1{}^2 q_1{}^2/(1 - \beta_1 q_1)\}$$

$$Y = E(y) + {}^1/_{q_1}$$

Where;
q_1 is traffic flow from major road (veh/s); q_2 is maximum traffic flow from midblock lane
β_1 minimum time headway between vehicles on major through road
β_2 minimum time headway between vehicles emerging from midblock U-turn lane
α average gap in the major road stream

In other to determine the number of vehicles waiting at the exit lane when the delay time per vehicle is known and Q is the mean rate of departure of vehicles, then;

$$E(n) = \frac{q_2}{Q}(1 - \frac{q_2}{Q})/(1 - \frac{q_2}{Q}) \tag{13}$$

Steady state and deterministic theories have been used in previous studies. However, Kimber and Hollis [6] developed an expression for queue length and delay that combined both steady-state and deterministic theory shown below.

Delay per unit time, $D = \frac{1}{2}\{(F^2 + G)^{1/2} - F\}$ (14)

Where;

$$F = \frac{(1 - \rho)(Qt)^2 - 2(Lo - 1)Qt - 4(1 - C)(Lo + \rho Qt)}{2\{Qt + 2(1 - C)\}}$$

$$G = \frac{2(Lo + \rho Qt)[Qt - (1 - C)(2Lo + \rho Qt)]}{Qt + 2(1 - C)}$$

And, Queue length, $L = \frac{1}{2}\{(A^2 + B)^{1/2} - A\}$ (15)

$$A = \frac{(1 - \rho)(Qt)^2 + 2(1 - Lo)Qt - 2(1 - C)(Lo + \rho Qt)}{Qt + (1 - C)}$$

$$B = \frac{4(Lo + \rho Qt)\{Qt - (1 - C)(Lo + \rho Qt)\}}{Qt + (1 - C)}$$

For;

C = 1 for random arrivals and service
SSD (X_s)C = 0 for regular arrivals and service
Q = capacity; $\rho = q/\mu$; q = demand flow;
t = time interval (t=0.25 if time interval is 15mins)
Lo = length of queue at start of time arrival

In any case an analytical approach to delay is used in the paper. An indication of likely values of β_1 (1s) β_2 (3s) and that of α varies between 4s and 12s according to intersection layout and speed. The computer program (PICADY) [7] developed by Transport Research Laboratory, England is based on deterministic approach. Delays are estimated from queuing theory.

3. SETUP OF IMPACT STUDY

The following criteria were used to define a roadway segment for the study: peak traffic volume exceeding 700 veh/hr/ln, speed limit between 60-90km/h, no direct access from abutting properties, no parking, two carriageway lanes in each direction. Federal Highway FT001 Senai, Kulai and FT005 Skudai, Johor Bahru, have been selected for the studies after careful strategic considerations. The setup of midblock U-turn impact study is illustrated below in figure 2. 24hr traffic volume, speeds, vehicle types, headways and gaps were recorded continuously for 8 weeks for both directions. Over 500,000 vehicles per roadway direction were captured on the data logger. Study was carried out under dry weather and daylight conditions. Note that free-flow volume and speed were taken as distance greater than SSD in other to minimise the influence of midblock U-turn.

Where;

L_1 = Outer Lane; L_2 = Overtaking Lane; SSD = minimum stopping distance, t = perception time (assume 2.5s), v = approach speed; a = deceleration time; D = Decision zone; W = Road width

$$D = X_o + W \tag{16}$$

$$X_s = vt + \frac{v^2}{2a} \tag{17}$$

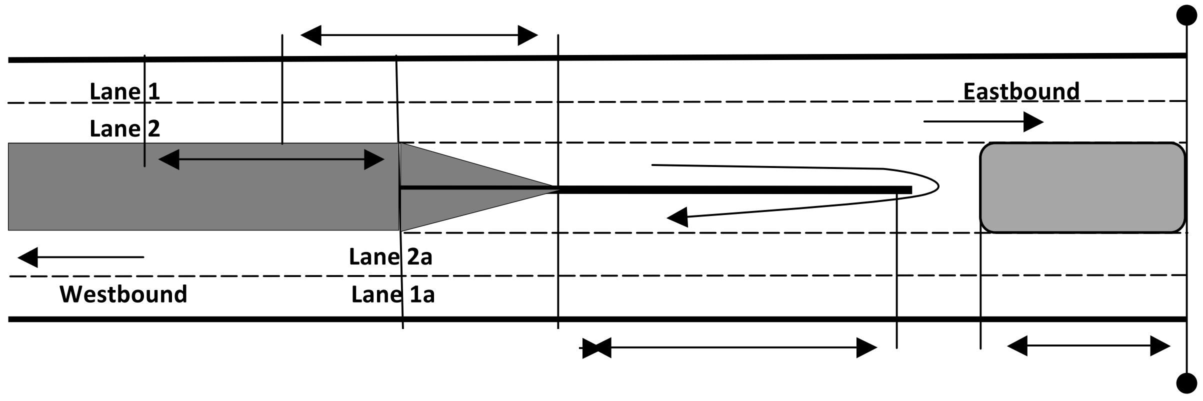

Figure 2 Typical layout of Survey Site

4. RESULTS AND DISCUSSION

Analysis of sample survey results is illustrated below using a step wise procedure:

Step1: Aggregated traffic data were disseminated and fitted into peak and off peak period under day light and dry weather conditions. Peak data were used for further analysis. Traffic volumes were converted into flows using appropriate passenger car units.

Step 2: Traffic flows and their corresponding speeds were used to derive densities. Speeds/densities per carriageway lane were related; for example at site 001:

$$U = -0.4203k + 66.17 \qquad R^2 = 0.98 \qquad (17)$$

Step 3: Determine the threshold capacity function under free-flow conditions, note that a threshold capacity is needed in other to establish whether flowrate contraction has occurred; for example, site 001

$$q = 66.17k - 0.4203k^2 \qquad R^2 = 0.98 \qquad (18)$$

Step 4: Test equation 17 and 18 for validity, then determine the vertex of equation 18;

$\partial q/\partial k = 66.17 - 0.8406k = 0$, hence, $k_o = 79$veh/km; and Q = 2604pcu/h/lane

The coefficient of determination (R^2) is greater than 0.5 suggesting that the equation is useful for modeling.

Step 5: Repeat steps 3 and 4 for carriageway lanes affected by midblock U-turn movements

Step 6: Estimate traffic shockwave as illustrated below in table1; for example, site 001:

Table 1: Traffic Kinematics Using Modified PCE

Drivers' Behaviour	Site	q_1 pcu/h/ln	k_1 veh/km	Q pcu/h	Q_2 pcu/h/ln	k_2 veh/km	q_1-Q_2	k_1-k_2	v_w Km/h
Deceleration & Diverging	001	985	21	2604	1471	45	-486	-69	0
	003	933	33	2509	1043	86	-110	-53	2
Acceleration & Merging	002	801	21	2353	1684	66	-883	-45	20
	004	755	39	2098	1263	72	-508	-33	15

Note: v_w – shockwave; Site pair (001 & 002) (003 & 004) indicate 2 direction flow

Step 7: Determine delay to vehicles traversing midblock U-turn dedicated lane; based on equation 12, average delay to midblock U-turn lane; $\overline{w_2} = 8.55s/veh$

Where;
q_1 is traffic flow from major road =0.223 veh/s
β_1 minimum time headway between vehicles on major road =1.53s
β_2 minimum time headway between vehicles on minor road =3.1s
α average gap in the major road stream = 5.51s
q_2 is maximum traffic flow from midblock lane =0.0875 or 5.25pce/min or 315pce/hr using equation 11.
Q = 1/8.55 ≈ 0.12

The number of vehicles waiting at the census point is;

$$E(n) = \frac{\frac{0.0875}{0.12}\left(1 - \frac{0.0875}{2x0.12}\right)}{\left(1 - \frac{0.0875}{2x0.12}\right)} = 0.73 \cong 1$$

As shown above in table 1, kinematics of traffic flow suggest that weaving of vehicles at decision zone area has not led to shockwave partly because drivers following the lead vehicle are able appraise traffic stream and control mechanism positively. Hence, traffic shockwaves along road section with diverging and deceleration lane are not substantial and critical, although vehicles may have difficulty in overtaking and weaving because of deceleration effect. Notwithstanding, relative speed reductions have not been abrupt not excessive as shown in table 1.

Acceleration and merging is a deft manoeuvre. Merging is more difficult than diverging because the through traffic flows are traversing along the faster lane. It is often a very dangerous manoeuvre that can trigger road accident. This is so because drivers along the overtaking lane are forced to either abandon the overtaking move in other to avoid collusion or ignore the risk altogether. Through traffic flows have priority in the conflict sections, and vehicles attempting to enter the stream can only do so during larger gaps of successive vehicles in the fast lane. In any case critical gap which is threshold by which merging stream judge whether to accept a gap has effect on traffic shockwaves. If the gap is larger than the critical gap, drivers accept it and enter the through traffic; otherwise drivers reject the gap and wait for the next gap. Since this is not a priority-controlled intersection the rule of critical gap fixed values or distribution does not apply. It's up to drivers to get the timing right. It can be argued that shockwaves are dependent on driver behaviour and the changing speed of the lead vehicle. This probably explains why shockwaves have different velocity values as shown in table 1.

The average delay to vehicles along dedicated midblock U-turn lane is 8.55s per vehicle and the maximum flow is about 315 pce/hr. Considering that where merging and cross manoeuvre take place, a potential if not actual collision may occur between vehicle and also given that the minimum headway between vehicles is 3.1s, delay of 8.5s per vehicle is not substantial. The number of vehicle waiting at the exit lane to join the through traffic flow is one. Therefore, it can be averred that traffic kinematic occasioned by midblock U-turning movement is appropriate; however traffic safety at exit from the dedicated lane remains a key concern for both service providers and road users.

5. CONCLUSION

As shown in the paper traffic kinematic can trigger shockwave and well as delay. Based on the hypothesis that midblock U-turn facilities at highway segments will cause shockwave and inherent delay at the turning lane, impact study was carried out in Skudai, Malaysia during daylight and dry weather conditions. On approach to the midblock U-turn, weaving and diverging between drivers did not cause kinematic shockwaves; whereas at merging section of the facility, gap acceptance caused delays and associated shockwaves in cases where drivers misjudge gaps. Consequently, the paper concluded that:

- Midblock U-turn facilities will induce traffic shockwave of about 20km/h at the exit lane
- In spite of weaving at decision zone, there is no evidence in the paper to suggest that midblock U-turn facilities can be called upon to account for traffic kinematics shockwave on approach
- Typical delay associated with midblock U-turn facilities of about 8s per vehicle can be expected at exit lane
- The hypothesis that midblock U-turn movements will cause delay and kinematic shockwave at the exit lane is valid

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